Technical Comments

Comment on "A Comparison of Different Forms of Dirigible Equations of Motion"

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PRETTY and Hookway[1] discuss a seeming paradox in the different forms for the equations of motion of a dirigible or submarine. They cite two forms of the equations of motion from two different references and state: "No studies that resolve the differences...have been found."

The equations for the motion of a solid through a fluid are well known[2] in classical hydrodynamics and give a unique and unambiguous answer to the question raised by Pretty and Hookway. For plane motion of a solid of revolution, the kinetic energy of the fluid plus solid may be represented by

$$2T=(A_f+M_s)u^2+(B_f+M_s)v^2+(Q_f+I_s)\dot{\theta}^2 \qquad (1)$$

Here M_s and I_s are the mass and moment of inertia of the solid, respectively. The velocities u and v refer to the direction of the principal axes fixed in the body and passing through its centroid, while $\dot{\theta}$ is the velocity of rotation of these axes. The velocity u is taken to be in the direction of the long axis. The effects of the fluid forces are represented by the longitudinal apparent mass A_f, the lateral apparent mass B_f, and the apparent moment of inertia Q_f.

By the usual methods of Lagrangian mechanics, a system with this expression for kinetic energy is found to have the equations of motion[2]

$$(A_f+M_s)\frac{du}{dt}=(M_s+B_f)\dot{\theta}v \qquad (2)$$

$$(B_f+M_s)\frac{dv}{dt}=-(M_s+A_f)\dot{\theta}u \qquad (3)$$

$$(Q_f+I_s)\frac{d\dot{\theta}}{dt}=(A_f-B_f)uv \qquad (4)$$

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Results substantially equivalent to the latter two equations are also given in somewhat different notation in Ref. 3 as explicitly applied to dirigibles.

The second of these equations, which deals with the transverse motion of the body of revolution, should correspond to Eqs. (1) and (2) of Ref. 1 except for terms in C_y arising from stabilizing and control surfaces and terms in m_g arising from the mass of the lifting gas. It is obvious by inspection where the difficulty lies in the conflicting equations of Ref. 1. It stems from the fact that the longitudinal and lateral apparent masses, A_f and B_f respectively, are different, while only a single apparent mass "m_{a_2}" appears in either Eq. (1) or Eq. (2) of Ref. 1. Eliminating the C_y and m_g terms and transposing to common form and notation, we have for the two cases in Ref. 1:

$$(m_{a_2}+M_s)\frac{dv}{dt}=-(m_{a_2}+M_s)\dot{\theta}u \qquad (5)$$

and

$$(m_{a_2}+M_s)\frac{dv}{dt}=-M_s\dot{\theta}u \qquad (6)$$

Taking into account the fact that the longitudinal apparent mass A_f is much smaller than the lateral apparent mass B_f, it is clear that, if "m_{a_2}" is equated to the lateral apparent mass B_f, then Eq. (6), which entirely omits the "m_{a_2}" terms on the right-hand side, is much more nearly correct than Eq. (5) in which m_{a_2} $(=B_f)$, the lateral apparent mass, is used in place of the much smaller longitudinal apparent mass A_f on the right-hand side.

This conclusion is in accord with what Pretty and Hookway conclude on the basis of comparisons of numerical solutions. Their contention that "the apparent masses and inertias are really aerodynamic force and moment acceleration derivatives" and should be treated that way, as exemplified in their Table 1, is entirely irrelevant to the solution of the problem they discussed. Whether the apparent mass and inertia terms (as long as they are correctly deduced) are put in one set of terms or another in the equations of motion is largely a matter of taste and "bookkeeping" and cannot affect the mathematical results.

References

[1] Pretty, J.R. and Hookway, R.O., "A Comparison of Different Forms of Dirigible Equations of Motion," *Journal of Guidance and Control,* Vol. 2, No. 2, March-April 1979, pp. 154-156.

[2] Lamb, H., *Hydrodynamics,* Dover Publications, New York, 1945, pp. 172-174.

[3] Arnstein, K. and Klemperer, W., "Performance of Airships," Division R, *Aerodynamic Theory,* Vol. VI, edited by W.F. Durand, Durand Reprinting Comm., 1943, pp. 110-112.